The Efficiency of Turbocharger Compressors With Diabatic Flows

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In most compressors the flow is adiabatic, but in low-speed turbochargers, the compression process has both heat transfer and work input. This paper examines different compressor efficiency definitions for such diabatic flows. Fundamental flaws in the use of the isentropic efficiency for this purpose are identified, whereas the polytropic efficiency can be used with or without heat transfer without ambiguities. The advantage of the polytropic approach for a practical application is demonstrated by analyzing the heat transfer in a turbocharger compressor. A simple model of the heat transfer allows a correction for this effect on the polytropic efficiency at low-speed to be derived. Compressor characteristics that have been corrected for this surprisingly large effect maintain a much higher efficiency down to low-speeds. [DOI: 10.1115/1.4000300]

1 Introduction

The operation of a turbocharger is not adiabatic, as heat is transferred from the hot turbine to the cold compressor. At high-speeds the heat transfer is negligible compared with the work transfer, but at low-speeds it has to be accounted for to determine the true efficiency of the compressor. The low-speed turbocharger belongs to a class of turbomachinery where heat transfer has to be considered. The most important of these are cooled gas turbine stages [1], intercooled compressors [2], and many small-scale turbomachinery applications [3].

In compressor tests, the absorbed power is usually determined by measurement of the temperature rise. The flow is taken as adiabatic so that the enthalpy rise is related purely to work input. To minimize any errors, compressor test rigs have to be insulated, and a certain time is needed before each measurement point is taken so that the unsteady heat flows are in equilibrium (see test procedures in ASME PTC10 [4]).

Most text books describe efficiency definitions for analysis of turbomachinery where the heat flow is negligible, but efficiency definitions for a diabatic flow are usually not mentioned. Several recent technical papers have examined this; see Sec. 2, whereby most of them have used an isentropic analysis. It is shown in this paper that the classical isentropic efficiency has severe deficiencies (or is completely flawed) when dealing with diabatic flows. This problem has prompted this publication whose main objective is to demonstrate the advantages of the polytropic efficiency for a diabatic process in a compressor. The conclusions are also valid for turbines, but no details of the turbine analysis are given.

The polytropic analysis given here is based on that of Traupel [5] and Dibelius [6]. The polytropic efficiency allows efficiency definitions for an adiabatic flow to be extended to those occurring with diabatic flows. As a practical example of this approach, it is shown that the polytropic analysis provides a novel means to estimate the amount of heat transfer between the turbine and the compressor of a turbocharger at low-speeds. Some of the analysis reported here was carried out by Fesich as a student project in the University of Stuttgart [7]. This paper is a much abbreviated version of a conference paper on this subject [8].

2 Diabatic Flows in Compressors

This section gives an overview of recent literature on the effect of heat transfer on the efficiency of compressors. At the outset it is useful to quote from a dictionary. The term “adiabatic” literally means impassable (from the Greek for “not-through-to-pass”), corresponding to a lack of heat transfer. Conversely, a process that includes heat transfer is generally called diabatic. One can use “nonadiabatic” to describe this, but this is actually a double negative for “diabatic.”

Diabatic flows in turbocharger compressors result from the high temperature gradient between the compressor and the other components of the turbocharger, and involve heat flows by radiation, conduction, and convection. The heat transfer to the compressor causes an apparent decrease in the efficiency due to the additional temperature rise, when compared with the temperature rise if the flow is assumed to be adiabatic. In addition, as the gas becomes warmer, more work is required for the next incremental increase in pressure, the so-called reheat effect as explained fully by Dixon [9].

Diabatic operation is important in microscale compressors with a high ratio of surface area to volume. Sirakov et al. [10] and Gong et al. [11] characterized diabatic performance of a microcompressor by including a heat addition term in the isentropic efficiency. The major assumption is that the heat transfer occurs in the inlet of the compressor and causes a preheating of the air before it is compressed. The compressor is still considered to be adiabatic.

Shaaban and Seume [12] took a similar approach for the heat transfer in a turbocharger compressor. In their work the heat transfer to the compressor is divided into two parts, that which takes place before and that after the compression process. The compressor itself is regarded as adiabatic and is characterized by means of an isentropic efficiency. The heat transfer to a turbocharger compressor has been examined by Bohn et al. [13] using a three-dimensional numerical model with conjugate heat transfer. Their simplified model for the heat fluxes takes into account heat addition to the compressor in the inlet, but allows heat to be lost from the compressor at the outlet.

Van den Braembussche [3] developed a prediction model for the effect of heat transfer on the polytropic efficiency of microcompressors for small gas turbines, assuming that the heat transfer varies linearly with the enthalpy increase in the compressor. This model differs from those mentioned earlier as it assumes that all the heat transfer takes place within the impeller and makes use of polytropic efficiency. It differs from the approach adopted here in that it assumes that the level of heat transfer varies proportional to the enthalpy change rather than being proportional to the flow work of the impeller.

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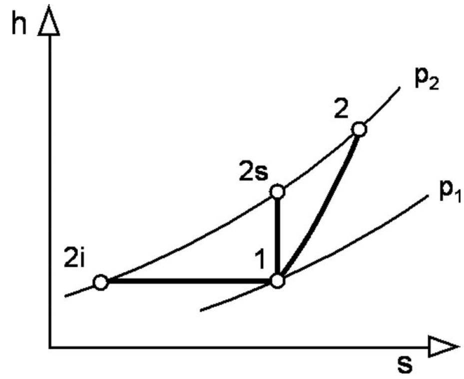

Fig. 1 Compressor efficiency definitions in Eq. (1). 1-2*i*: isothermal ($x=2i$), 1-2*s*: isentropic ($x=2s$), and 1-2: polytropic ($x=2$).

3 Diabatic Efficiency Analysis

We consider a compression process between the inlet state (1) and outlet state (2) of a steady flow in a compressor. The compressor efficiency is then defined as the work that would be required by a perfect compressor undergoing some reference process compared with the actual work expended, the idea being that a perfect compressor undergoing a hypothetical ideal process would require less shaft power than a real compressor with frictional dissipation. The actual work is invariably considered as the shaft power or the specific shaft work, but more difficulty occurs with the definition of a reference process to define what is perfect. There are generally three competing reference processes that can be used to define the perfect compressor: an isentropic process between the inlet and outlet pressures, a polytropic process between the actual inlet and outlet conditions, or an isothermal process between the same inlet and outlet pressures, see Fig. 1.

If for clarity we neglect any kinetic energy change across the compressor, and consider only internal losses, the general definition of static-static compressor efficiency for all three ideal processes can then be stated as

$$\eta = \frac{\int_1^x v dp}{w_{t,12}} \tag{1}$$

The denominator is the actual specific shaft work input and the numerator is the specific flow work (or vdp work) required to produce the pressure rise of the chosen ideal process. Note that the notation flow work used here is consistent with continental European usage of this term, which is also sometimes known as the polytropic head, but that in some American and English thermodynamic textbooks the name flow work is given to the difference between the pv products at inlet and outlet of an open system.

The difference between the isentropic, polytropic, and isothermal processes relates to the different integration paths used for the integration from state 1 to condition x, as shown in Fig. 1. The polytropic process determines the polytropic work by integrating along the actual compression path from inlet 1 to outlet 2, the isentropic process considers an ideal reference process at constant entropy between 1 and a virtual state $2s$, and the isothermal process between 1 and $2i$ considers a diabatic process with just sufficient continuous heat removal that the gas temperature remains constant at the inlet value throughout the compression process. Because of the divergence of the constant pressure lines in the h-s diagram the isothermal process actually requires less work than the ideal isentropic process. The isothermal process is always diabatic with heat removal, the isentropic process is usually considered to be adiabatic and reversible, and the polytropic process may be diabatic or adiabatic and reversible or irreversible.

3.1 Analysis Using the Isentropic Efficiency. At the outset it needs to be firmly stated that the use of an isentropic process as a reference process to describe a reversible diabatic flow is clearly fundamentally flawed, as a reversible diabatic flow does not have constant entropy.

For the isentropic process state, x in Eq. (1) is the virtual or hypothetical state $2s$, which has the same outlet pressure as the real process but the same entropy as at the inlet state. The second law formulated in terms of the Gibbs equation is

$$dh = vdp + Tds \tag{2}$$

It is seen that the enthalpy change in an adiabatic compression process, with no entropy change due to heat transfer, is the sum of the flow work (vdp) and the dissipation (Tds). The isentropic process then describes the ideal work of a perfect adiabatic machine with no dissipation losses and no change in entropy.

For a constant entropy process we integrate Eq. (2) to

$$h_{2s} - h_1 = \int_1^{2s} vdp \tag{3}$$

The combined work and heat transfer of the process are defined by the first law for steady flow through an open system as

$$w_{t,12} + q_{12} = h_2 - h_1 + \tfrac{1}{2}(c_2^2 - c_1^2) \tag{4}$$

If we have an adiabatic flow then the specific work input is the same as the change in enthalpy (for clarity we neglect the changes in kinetic energy). In this way we determine the usual expression for the isentropic efficiency in an adiabatic flow as

$$\eta_s = \frac{\Delta h_s}{\Delta h} = \frac{\int_1^{2s} vdp}{h_2 - h_1} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} \tag{5}$$

which is often called the adiabatic efficiency.

A first problem with this equation for a diabatic flow is that the shaft power is no longer equal to the change in enthalpy, so the denominator of Eq. (1) is actually wrong in Eq. (5) for a diabatic flow. The measured enthalpy and temperature rise overestimate the actual work input due to the warming caused by the heat addition, and this needs to be subtracted to get a proper measure of the diabatic isentropic efficiency. Several authors have tried to make a correction for this effect. If we neglect the kinetic energy terms we obtain from Eqs. (1), (3), and (4) that for a flow with heat transfer the "diabatic isentropic efficiency" would be given by

$$\eta_{sq} = \frac{h_{2s} - h_1}{(h_2 - h_1) - q_{12}} = \frac{T_{2s} - T_1}{(T_2 - T_1) - q_{12}/c_p}$$
$$\eta_{sq} = \frac{1}{\dfrac{1}{\eta_s} - \dfrac{q_{12}}{c_p(T_{2s} - T_1)}} = \frac{1}{\dfrac{1}{\eta_s} - \dfrac{q_{12}}{h_{2s} - h_1}} \tag{6}$$

This particular form has been chosen for comparison with a later equation in the paper. The correction to the efficiency is related to the amount of heat transfer in relation to the enthalpy rise in the ideal isentropic process, a result that has already been given by other authors mentioned above.

A second problem in Eq. (5) arises because a reversible diabatic flow (with no dissipation losses) is not isentropic, so there is no justification for using an isentropic process as a reference for the ideal work required by a perfect diabatic compressor. A process with the same heat transfer as the real process but with no dissipation losses should logically be used, but this is not isentropic. Moreover, if heat addition to the compressor takes place then the ideal isentropic process with heat addition would require the dissipation losses to be negative to counteract the entropy rise due to the heat added. This clearly violates the second law and is a weak basis for the criterion of perfection to be used in a thermodynamic

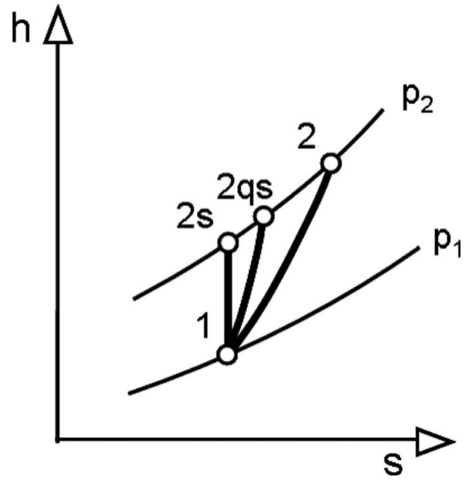

Fig. 2 Diabatic process and isentropic efficiency

efficiency definition.

The correct ideal comparison process would include the heat transfer, as shown in the process from 1 to $2qs$ in Fig. 2. The isentropic temperature change for the true work input of an isentropic process would lead to an equation of the form

$$\eta_{sq} = \frac{\int_1^{2qs} v dp}{(h_2 - h_1) - q_{12}} \tag{7}$$

Equation (7) has the practical difficulty that the integration along the path from state 1 to state $2qs$ is needed, so the heat transfer needs to be known. Like Eq. (6), however, it reverts to the adiabatic form (Eq. (5)) in a process with no heat transfer.

Clearly the commonly used isentropic efficiency is flawed for the analysis of a diabatic flow. It is probably for this reason that in many turbomachinery publications the term adiabatic efficiency is used instead of the term "isentropic" efficiency. In fact, what is generally meant in the use of Eq. (5) is a comparison process that is both adiabatic and reversible and so it should really be called the "adiabatic isentropic efficiency," as opposed to the "diabatic isentropic efficiency" implied by Eq. (6) given by several authors.

Several authors have nevertheless adapted the isentropic (or adiabatic) efficiency along the lines of Eq. (6) to analyze compressors with heat addition. The difficulty related to the inappropriate ideal process is generally avoided by splitting the diabatic compression process into different parts, as shown in Fig. 3. The heat transferred to the compressor is assumed to take place at constant

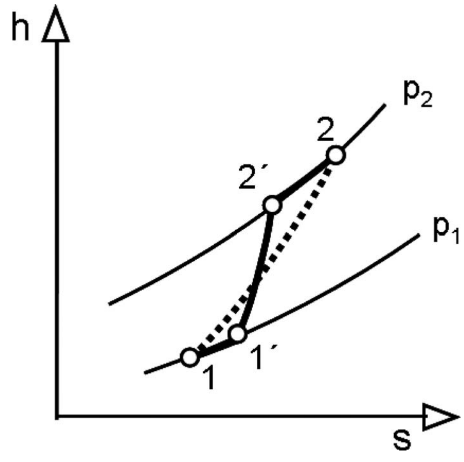

Fig. 3 Simplified *h-s* model for nonadiabatic compression with heat added before and after adiabatic compression

pressure and is divided into that which occurs before (1 to 1′) and that after (2′ to 2) the actual compression process. The compression process itself (1′ to 2′) is then still considered to be an adiabatic process with work input, so that Eq. (5) can be used with no ambiguity.

3.2 Analysis Using the Polytropic Efficiency. This section derives the equations for the polytropic analysis of a compression process with heat transfer. The difference between the polytropic analysis and the isentropic analysis is that the value of state x in the integral shown in Eq. (1) is the real state of the gas at the end of the compression process, state 2. The integration of the flow work (vdp) along the actual process path allows the analysis to take into account both heat transfer and dissipative processes in the fluid and needs no reference to an artificial virtual state ($2s$).

The second law, as the Gibbs equation in Eq. (2) above, can be integrated along the actual compression path to give

$$h_2 - h_1 = \int_1^2 v dp + \int_1^2 T ds = \int_1^2 v dp + \int_1^2 (T ds)_q + \int_1^2 (T ds)_{\text{irrev}}$$
$$= y_{12} + q_{12} + j_{12} \tag{8}$$

where y_{12} is the useful flow work produced by the process, q_{12} is the heat addition and relates to the entropy change (either positive or negative) due to the heat transfer across the flow boundaries, and j_{12} is the heat produced by the frictional losses or dissipation (and is always positive). Note that the enthalpy change is determined by the end states alone, but that the flow work term vdp and the entropy production terms Tds depend on the integration path between the inlet and the outlet. Moreover, the entropy production term includes both the entropy change due to the heat transfer to the process and the entropy change due to the irreversible frictional dissipation as

$$T ds = (T ds)_q + (T ds)_{\text{irrev}} = dq + dj \tag{9}$$

Equation (8) can be written in combination with the first law as

$$w_{t,12} = y_{12} + j_{12} + \tfrac{1}{2}(c_2^2 - c_1^2) \tag{10}$$

This identifies that the work input to the compressor appears as flow work that causes a useful increase in pressure (given by the polytropic head rise y_{12}), as frictional dissipation j_{12} and as a rise in the kinetic energy. Any heat transfer to the compressor does not actually appear explicitly in Eq. (10). The amount of heat transfer influences the integration path from state 1 to state 2 and so influences both the flow work and the dissipation.

A rational definition of efficiency for a compressor in a diabatic flow, including the kinetic energy differences for completeness, is then

$$\eta_p = \frac{y_{12} + \frac{1}{2}(c_2^2 - c_1^2)}{w_{t,12}} = 1 - \frac{j_{12}}{w_{t,12}} \tag{11}$$

where the useful pressure rise and associated change in kinetic energy are expressed as a fraction of the shaft work. Note that this equation is valid for both an adiabatic and a diabatic process, as the heat transfer does not appear explicitly in Eq. (10). If we neglect the kinetic energy terms then this equation becomes the same as Eq. (1) for the polytropic efficiency in an adiabatic flow.

This equation has the enormous advantage that the role of the frictional dissipation is made immediately clear. The common engineering use of "$1-\eta$" as a convenient statement of the losses is more or less self-evident. In this way, the losses are simply the energy dissipation as a fraction of the shaft work supplied, with or without heat transfer, as follows:

$$1 - \eta_p = \frac{j_{12}}{w_{t,12}} = \frac{\int_1^2 (T ds)_{\text{irrev}}}{w_{t,12}} \tag{12}$$

The flow work and entropy change terms in Eq. (8) require integration along the actual flow path. In some cases information from experiments or simulations may be available and intermediate states of the real process may be known to allow integration along the real path of the process. In general, however, no detailed information is available about the intermediate states in the machine, so the real process is approximated by a replacement polytropic process for this integration. The polytropic process has the same end states as the real process and is defined as one in which each differential step in the process has a constant value of the polytropic ratio

$$\upsilon = \frac{dh}{v dp} = 1 + \frac{Tds}{v dp} = 1 + \frac{dq + dj}{dy} \tag{13}$$

The enthalpy change, heat transfer, and dissipation are in the same ratio to the flow work for each step of the process. (The symbol for the polytropic ratio is taken as υ to remain consistent with Traupel [5], but the reader should be aware that this symbol is not the specific volume v). Integrating this for the whole process with a constant polytropic ratio leads to

$$\upsilon = \frac{h_2 - h_1}{y_{12}} = 1 + \frac{q_{12} + j_{12}}{y_{12}} \tag{14}$$

The same polytropic ratio can be attained by means of different amounts of dissipation and heat transfer.

The more usual description of a polytropic process is given as one in which the polytropic exponent n, also known as the polytropic index, in the equation

$$pv^n = \text{const} \tag{15}$$

is constant along the compression path. For an ideal gas with

$$pv = RT, \quad h = c_p T \tag{16}$$

it can be shown by use of the Gibbs equation that

$$\upsilon = \frac{(n-1)/n}{R/c_p} = \frac{(n-1)/n}{(\gamma-1)/\gamma} \tag{17}$$

Equation (17) shows that a process with a constant polytropic ratio also has a constant polytropic exponent. It should be noted that the isentropic exponent γ in these equations is not due to the influence of some isentropic process, but simply to the fact that for an ideal gas $R/c_p = (\gamma-1)/\gamma$.

We now consider an infinitesimal compression process from p to $p+dp$ with both energy dissipation and heat transfer. If this process were reversible and adiabatic the Gibbs equations show that it would require a work input of

$$dh = dy = v dp \tag{18}$$

For an irreversible process in a compressor more work is required, and this can be characterized by the small-scale polytropic efficiency, so that the shaft work becomes

$$dw_t = \frac{1}{\eta_p} v dp \tag{19}$$

The dissipated energy is the difference between these equations

$$dj = (Tds)_{\text{irrev}} = \left(\frac{1}{\eta_p} - 1\right) v dp = \left(\frac{1}{\eta_p} - 1\right) dy \tag{20}$$

so that in an adiabatic flow the dissipation can be defined as a fraction of the flow work, in terms of the polytropic efficiency.

If we consider a process with heat transfer alone, we can define the heat transfer as a fraction of the flow work, as

$$dq = (Tds)_{\text{rev}} = \zeta_q v dp = \zeta_q dy \tag{21}$$

For the situation with combined heat transfer and work input we can substitute these equations into Eq. (13), and this leads to the following equation for the polytropic ratio:

Table 1 Polytropic processes with heat and work transfer

Case	Exponent	Process	Equation	Polytropic ratio υ $\upsilon = \zeta_q + 1/\eta_p$
a	$n=1$	Isotherm	$pv=\text{const}$	$\upsilon=0$
b	$1<n<\gamma$			$0<\upsilon<1$
c	$n=\gamma$	Isentrope	$pv^\gamma=\text{const}$	$\upsilon=1$
d	$\gamma<n<\infty$			$1<\upsilon<\gamma/(\gamma-1)$
e	$n=\pm\infty$	Isochor	$v=\text{const}$	$\upsilon=\gamma/(\gamma-1)$
f	$-\infty<n<0$			$\gamma/(\gamma-1)<\upsilon<\infty$
g	$n=0$	Isobar	$p=\text{const}$	$\upsilon=\infty$

$$\upsilon = \frac{(n-1)/n}{(\gamma-1)/\gamma} = \zeta_q + \frac{1}{\eta_p} \tag{22}$$

In the polytropic analysis of turbomachines both the heat transfer and the dissipation processes are considered together through the fact that both cause a change in entropy. This is not possible in the isentropic analysis because the entropy is taken as constant in the ideal comparison process used as a reference.

3.3 Special Cases of the Polytropic Analysis. Some special cases of these equations are illuminating. A summary of these is given in Table 1 and these are sketched graphically in Fig. 4, which is an extension to diabatic flows of a figure given by Cordes [14]. The cases denoted by (a)–(d) involve compression with a density increase, case (e) is constant density, and cases (f) and (g) are expansion processes with a density decrease. These latter cases are, of course, seldom relevant in compressors but do occur at extreme operating points on the low pressure part of the characteristics.

First we consider an adiabatic process with no heat transfer. The equations given above reduce to the standard expressions for the polytropic efficiency of a compressor

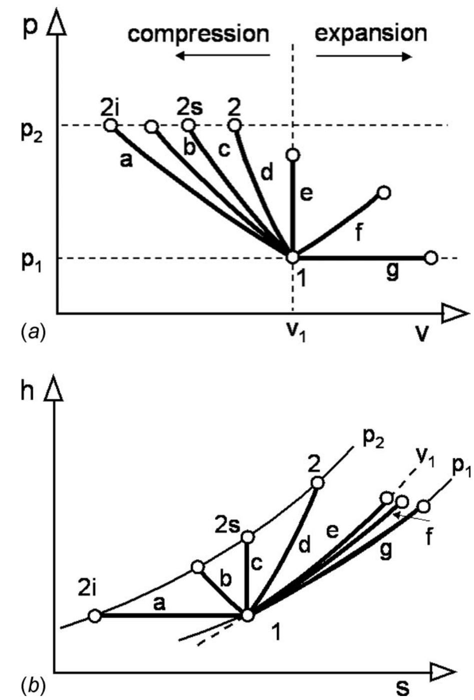

Fig. 4 (*a*) *p-v* **diagram of processes in Table 1. (*b*) *h-s* diagram of processes in Table 1.**

$$\zeta_q = 0, \quad \eta_p = \frac{1}{\upsilon} = \frac{\gamma - 1}{\gamma}\frac{n}{n-1} \tag{23}$$

For all real adiabatic processes with $\eta_p < 1$ the exponent n is larger than the isentropic exponent and the polytropic ratio is $\upsilon > 1$. If we take the polytropic efficiency to be unity then we get the equations for a reversible adiabatic process, so that the isentropic exponent is the same as the polytropic exponent.

$$\zeta_q = 0, \quad \eta_p = \upsilon = 1, \quad n = \gamma \tag{24}$$

It is this ideal reversible adiabatic polytropic process that is used to define the flow work in Eq. (1) when considering the isentropic efficiency, so that in an adiabatic process with no losses the polytropic and isentropic processes are the same.

Another issue is the use of these equations to model off-design operating points, for example, a process with zero efficiency and a pressure ratio of unity. The work input is then all dissipated or appears as kinetic energy, such that no pressure rise occurs at all. This process would be at constant pressure and for this process the polytropic exponent is zero.

$$\zeta_q = 0, \quad \eta_p = 0, \quad \upsilon = \pm\infty, \quad n = 0 \tag{25}$$

For diabatic processes the heat transfer coefficient is not zero and the effective polytropic exponent then depends on both heat transfer and energy dissipation, as given in Eq. (22). If the amount of heat addition were to be exactly the same as the actual dissipative warming in a real adiabatic process, then the diabatic process with no losses would have the same polytropic ratio and polytropic exponent as the adiabatic process with losses. In terms of their end states they would be indistinguishable. Thermodynamically we can model an irreversible adiabatic polytropic process by a reversible polytropic process with heat transfer. The special feature of the polytropic analysis, which makes it so useful, is that heat transfer and energy dissipation are interchangeable in this way.

Another interesting special case for diabatic processes is one in which the heat removed from the process exactly cancels the heat produced by dissipation such that there is no change in specific entropy and the polytropic exponent is then the same as the isentropic exponent, with

$$\zeta_q = 1 - \frac{1}{\eta_p}, \quad \upsilon = 1, \quad n = \gamma \tag{26}$$

This process would be isentropic, but is neither adiabatic nor reversible.

A further interesting case is the isothermal compression already shown in Fig. 1, where all the energy from the work input and frictional dissipation are removed by heat transfer such that the temperature remains constant and we obtain

$$\zeta_q = -\frac{1}{\eta_p}, \quad \upsilon = 0, \quad n = 1 \tag{27}$$

For completeness, it should be noted that the polytropic equation sketched in the upper part of Fig. 4 is the polytropic process expressed in a p-υ diagram, using Eq. (15), and in the lower part the process is shown in the h-s diagram using

$$s_2 - s_1 = R\left(\frac{\gamma}{\gamma - 1} - \frac{n}{n-1}\right)\ln\left(\frac{h_2}{h_1}\right) \tag{28}$$

which is derived from Eq. (15) with the Gibbs equation.

3.4 Apparent Polytropic Efficiency of a Diabatic Process.

We now consider measurements of efficiency in a compressor with heat transfer where the amount of heat transfer is not precisely known or even incorrectly assumed to be nonexistent. The use of the polytropic analysis leads to the definition of an efficiency that is the apparent adiabatic polytropic efficiency. This is the typical compressor situation with heat addition and dissipation losses and corresponds to case (d) in Table 1. For this case, we can rewrite Eq. (22) as

$$\upsilon = \frac{(n-1)/n}{(\gamma - 1)/\gamma} = \zeta_q + \frac{1}{\eta_p} = \frac{1}{\eta_{pq}} \tag{29}$$

where η_{pq} could be called the "diabatic polytropic efficiency" or the apparent adiabatic polytropic efficiency in a flow with heat transfer. The test data on the assumption of no heat transfer would define a polytropic efficiency of η_{pq} but in reality the polytropic efficiency is η_p, and there is an additional heat transfer coefficient ζ_q, which is related by Eq. (29). Reformulation of this equation shows

$$\eta_p = \frac{1}{\dfrac{1}{\eta_{pq}} - \zeta_q} = \frac{1}{\dfrac{1}{\eta_{pq}} - \dfrac{q_{12}}{y_{12}}} \tag{30}$$

which provides an equation for the actual polytropic efficiency to be expected in an adiabatic flow. Note that the form of this equation is directly similar to Eq. (6) from the isentropic analysis, except that the heat transfer component appears as a ratio to the actual specific flow work rather than to the isentropic flow work. This equation confirms the observations of other authors using an isentropic analysis that the real polytropic efficiency is underestimated when heat transfer to the compressor occurs.

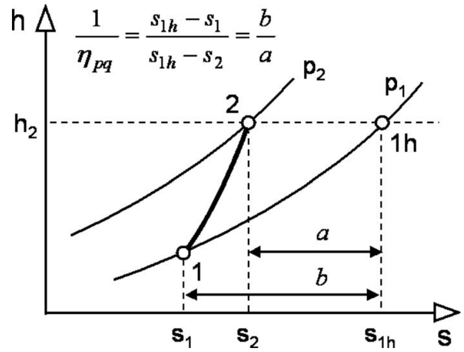

Fig. 5 Diabatic polytropic efficiency displayed in terms of entropy differences in an *h-s* diagram, as given in Eq. (31)

3.5 Comparison of Isentropic and Polytropic Analyses.

Most of the advantages of the polytropic over the isentropic efficiency for adiabatic flows are described by Traupel [5]. A key advantage of the polytropic efficiency is that in multistage machines no inconsistency arises over the small-scale and the global polytropic efficiency. The polytropic efficiency of a turbomachine made up of several stages, each with the same polytropic efficiency, has the same overall polytropic efficiency. This is not the case with the isentropic efficiency. More work is needed for each successive compression step with the same isentropic efficiency, leading to a lower overall isentropic efficiency for the multistage compression process (and a higher efficiency for expansion processes). To deal with this, the isentropic analysis needs to include an artificial "reheat factor," which is not required in the polytropic analysis. In fact, as shown by Dixon [9] among others, the reheat factor is simply the ratio of the polytropic and isentropic efficiencies. Because this factor differs for different duties, the isentropic efficiency actually has only a very limited usefulness, which is to compare machines that operate under closely similar conditions.

It is often stated that the disadvantage of the polytropic process as a comparison process is that it is not possible to visualize this in the h-s diagram, but Casey [15] showed that this is not the case for adiabatic flows. Figure 5 extends this approach to the case discussed here with heat transfer. The analysis follows closely that

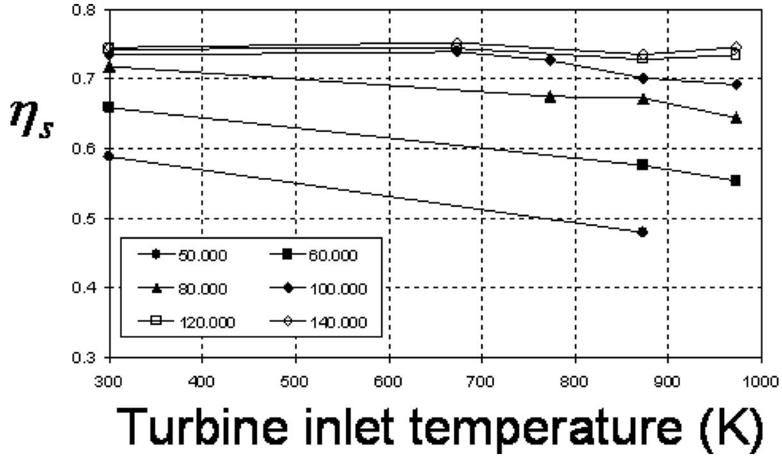

Fig. 6 Apparent isentropic efficiency of a turbocharger compressor stage with different turbine inlet temperatures and rotational speeds

given by Casey [15], and the resulting equation for the adiabatic polytropic efficiency in terms of entropy differences is

$$\zeta_q + \frac{1}{\eta_p} = \frac{1}{\eta_{pq}} = \frac{s_{1h} - s_1}{s_{1h} - s_2} = \frac{b}{a} \quad (31)$$

State $1h$ is the hypothetical state with the same pressure as at the inlet and the same enthalpy as at the outlet, which, in fact, is the case where all the work input is dissipated (as given in Eq. (25)). The diabatic polytropic efficiency, which includes the heat transfer coefficient and the adiabatic polytropic efficiency, can be seen to be the ratio of horizontal distances parallel to the entropy axis in the *h-s* diagram. The isentropic efficiency is, of course, easily visualized as the ratio of vertical distances parallel to the enthalpy axis.

4 Analysis of a Turbocharger Compressor With Heat Addition

In this section a practical application of the polytropic analysis for the diabatic compression process at low-speeds in a turbocharger compressor is described. This provides a useful demonstration of the many issues discussed above and demonstrates the utility of the polytropic analysis.

The adiabatic compressor efficiency represents the efficiency without heat transfer to the compressor. It can be measured by conducting "cold measurements," i.e., driving the turbine with compressed (cold) air instead of hot gases from an engine or a combustor in a test rig. The diabatic compressor efficiency is the efficiency measured under nonadiabatic operating conditions with heat transfer from the turbine.

Measurements of the compressor efficiency made at different speeds with different turbine inlet temperatures in a turbocharger test rig have been published by Berndt et al. [16]; see Fig. 6. These measurements show that for a particular design of a small turbocharger operating at a rotational speed of 140,000 rpm, corresponding to a pressure ratio of 1.8 and a tip-speed Mach number close to unity, the measured isentropic efficiency is found to drop only slightly with the turbine inlet temperature. At higher-speeds less effect can be expected, but at lower-speeds the turbine inlet temperature increasingly affects the compressor efficiency. At 80,000 rpm, corresponding to a low pressure ratio of 1.25 and a tip-speed Mach number of 0.65, an apparent drop in efficiency of around 1% for each 100°C increase in the turbine inlet temperature can be observed. At 50,000 rpm, which corresponds to a pressure ratio of 1.1 and a very low tip-speed Mach number of 0.45, the efficiency falls nearly 2% for each 100°C increase in the turbine inlet temperature. Measurements at lower-speeds are not available but would in any case be difficult to use as the accuracy of efficiency measurement at even lower-speeds would decrease with low temperature differences.

Interestingly, the same measurements show that the pressure rise is not at all affected by the temperature changes, so it is reasonable to assume that the efficiency deficit at low-speeds is entirely due to the heat transfer effect shown in Eq. (30). In these measurements the work input has been determined by temperature measurements, so it is clear that the fall in efficiency is accompanied by a similar increase in the apparent nondimensional enthalpy rise coefficient of the stage. A simple model for this is developed below.

For an adiabatic flow we can express the polytropic pressure rise coefficient in terms of polytropic efficiency and the enthalpy rise coefficient as follows:

$$\psi_p = \frac{\int_1^2 v\,dp}{u_2^2} = \frac{y_{12}}{u_2^2} = \frac{y_{12}}{w_{t12}}\frac{w_{t12}}{u_2^2} = \eta_p \lambda \quad (32)$$

In the diabatic flow we similarly obtain

$$\psi_{pq} = \eta_{pq} \lambda_q \quad (33)$$

The heat transfer has no effect on the pressure ratio, so the pressure rise coefficient is unchanged. An apparent increase in work input and enthalpy rise due to heat transfer effects are then directly related to an apparent decrease in the efficiency. We define the diabatic enthalpy rise coefficient as

$$\lambda_q = \frac{\Delta h_{t12}}{u_2^2} = \frac{w_{t12} + q_{12}}{u_2^2} = \lambda + \frac{q_{12}}{u_2^2} \quad (34)$$

which includes the enthalpy rise due to work and heat transfer. This corresponds to the measured enthalpy rise coefficient, and the difference to the adiabatic coefficient can be given as

$$\lambda_q - \lambda = \frac{q_{12}}{u_2^2} \quad (35)$$

The heat transfer coefficient can now be determined as

$$\zeta_q = \frac{q_{12}}{y_{12}} = \frac{(\lambda_q - \lambda)u_2^2}{\psi_p u_2^2} = \frac{(\lambda_q - \lambda)}{\psi_p} \quad (36)$$

The true polytropic efficiency of the stage without heat transfer can then be determined from that with heat transfer as follows:

$$\eta_p = \frac{1}{\dfrac{1}{\eta_{pq}} - \zeta_q} = \frac{1}{\dfrac{1}{\eta_{pq}} - \dfrac{(\lambda_q - \lambda)}{\psi_p}} \quad (37)$$

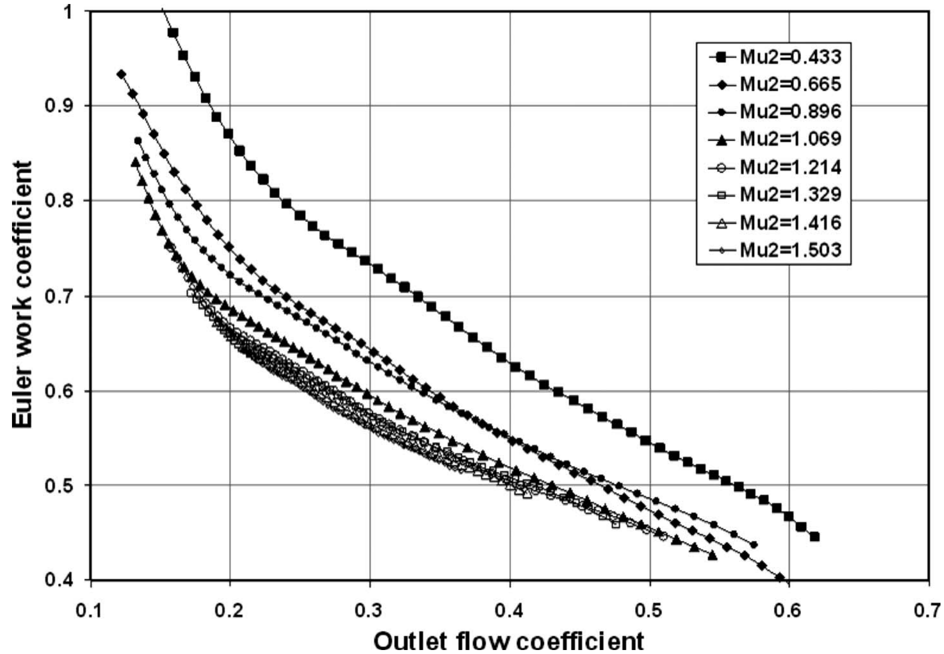

Fig. 7 Work coefficient versus outlet flow coefficient characteristic (derived from a measured performance map of a turbocharger compressor in Ref. [17])

4.1 Enthalpy Rise Coefficient of a Compressor Stage. The work input of a compressor stage can be explained with a simple one-dimensional analysis based on the Euler equation. For an adiabatic flow the enthalpy rise coefficient is

$$w_{t,12} = h_{t2} - h_{t1} \tag{38}$$

and the enthalpy rise coefficient of the impeller is

$$\lambda = \frac{h_{t2} - h_{t1}}{u_2^2} = \frac{w_{t,12}}{u_2^2} \tag{39}$$

Under the assumption that the flow has no swirl at the inlet of the impeller, then the Euler work due to the work input of the impeller can be shown from the velocity triangles to be related directly with the flow coefficient at the impeller outlet, the slip factor, and the impeller outlet blade angle, as follows:

$$\lambda_{\text{Euler}} = \frac{c_{u2}}{u_2} = 1 - \frac{c_s}{u_2} + \phi_2 \tan \beta_2' \tag{40}$$

Note that the impeller outlet angle is negative for a backswept impeller with the notation used here. For a constant value of the slip factor the work input coefficient can be expected to decrease linearly with the impeller outlet flow coefficient.

This equation includes the local flow coefficient at the impeller outlet and, as usually no measurements are available at the outlet plane of the impeller, this cannot be derived directly from the global stage measurements with no further assumptions. The real work input into the stage must also take account of the parasitic disk friction losses on the impeller disk so that the final equation for the work input coefficient is

$$\lambda = \left(1 + \frac{k}{\phi_{t1}}\right)\lambda_{\text{Euler}} \tag{41}$$

where k is an empirical constant representing the disk friction power. The value of k needs to be taken from values found on other similar impellers, or from suitable correlations of disk friction power, and typically $k=0.004$.

Casey and Schlegel [17] developed a technique to estimate the flow coefficient at the impeller outlet from measured performance characteristics based on an assumed level of performance of the vaneless diffuser system. This procedure allows the impeller Euler enthalpy rise versus flow coefficient relationship, as shown in Eq. (40), to be estimated from global measured characteristics.

The results of such an analysis are given in Fig. 7. This typical case, based on test data from a commercial turbocharger supplier, shows that the linear relationship between the enthalpy rise coefficient and the impeller outlet flow coefficient expected from the Euler equation is generally confirmed by the measurement data. Some discrepancies can be seen. At low flows the deviation from a linear relationship is believed to be due to inlet flow recirculation, which is not included in Eqs. (40) and (41). Of relevance here is the fact that the curves for lower tip-speed Mach numbers (all given dark symbols) show much higher values of the enthalpy rise coefficient than those at higher Mach numbers, which all have very similar values for a given flow coefficient. The discrepancy increases with lower tip speeds (as denoted by the tip-speed Mach number) and at lower flow coefficients, and is present, but small, at a tip-speed Mach number of unity. The apparent enthalpy rise measured in the impeller is higher than that based on the Euler equation due to the effect of the heat transfer on the temperature rise, as described by Eq. (35).

The difference between the enthalpy rise coefficient of the high-speed and low-speed characteristics is a direct measure of the specific heat transfer to the compressor and allows the amount of heat transfer to be approximately determined. It is unlikely that such an approach can be used for particularly accurate calculations. Firstly it relies on many assumptions, and secondly the accuracy of typical low-speed characteristics with regard to efficiency measurement is usually poor. Nevertheless this analysis allows a first estimate of the effect of the heat transfer to be obtained from the comparison of the high-speed and low-speed work input curves in a diagram similar to Fig. 7, based entirely on global measured stage characteristics.

4.2 Heat Transfer Model. We assume that at low-speed, where the compressor temperature rise is only moderate, the heat transfer rate from the hot parts of the turbocharger is constant for a given turbine inlet temperature. The specific heat flow then depends on the mass flow through the compressor. Using conventional nondimensional parameters we can derive that

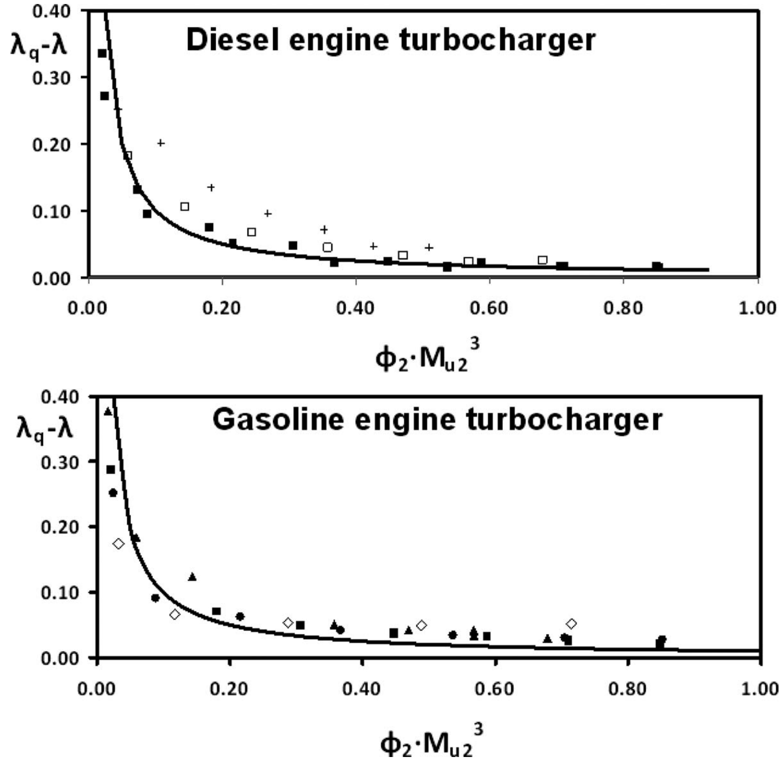

Fig. 8 Comparison of test data from a turbocharger test rig operating with a turbine inlet temperature of 600°C, for two turbochargers with the constant heat transfer model of Eq. (46) (k_q=0.01). The different symbols represent measurements at different speeds.

$$\dot{m} = \rho_2 \phi_2 u_2 \pi b_2 D_2 \tag{42}$$

and that the power input is

$$P = \dot{m} w_{t,12} = \dot{m} \lambda u_2^2 = \rho_2 \phi_2 \lambda u_2^3 \pi b_2 D_2 \tag{43}$$

The specific heat flow per unit mass flow can be expressed as

$$q_{12} = \dot{Q} / \rho_2 \phi_2 u_2 \pi b_2 D_2 \tag{44}$$

For a constant heat transfer rate we would expect the difference between the diabatic and the adiabatic work coefficients to be given by

$$\lambda_q - \lambda = \frac{q_{12}}{u_2^2} = \frac{\dot{Q}}{\rho_2 \phi_2 u_2^3 \pi b_2 D_2^2} \tag{45}$$

where the denominator is proportional to the power of the compressor. We can express this nondimensionally as

$$\lambda_q - \lambda \approx \frac{k_q}{\phi_2 M_{u2}^3} \tag{46}$$

where k_q is a dimensionless constant that depends on the rate of heat transfer per unit area.

The analysis suggests that with a constant heat transfer rate the difference between the diabatic and adiabatic work coefficients is inversely proportional to the product of the cube of the impeller tip-speed Mach number and the flow coefficient, and is related to the compressor power as shown in Eq. (43). The constant k_q depends on the design of the stage, the heat paths in the turbocharger, and the turbine temperature. In a typical turbocharger test with constant turbine inlet temperature at 600°C it will remain sensibly constant but will clearly change with the turbine inlet temperature if this changes.

Clearly such a simple approach cannot be expected to be particularly accurate as a model for the very complex heat flows in a turbocharger. Nevertheless, the analysis of the measured characteristics of two very different turbocharger compressor stages using this approach shows that the measurement data follow the trend given by Eq. (46), as shown for the two representative cases in Fig. 8. In both cases the same value of the constant k_q has been used, but insufficient data have been analyzed to confirm whether this is always similar. The scatter is high especially for the low-speed values, but the trends support the hypothesis of constant heat transfer.

Using these results for the heat transfer coefficient allows Eq. (37) to be used to predict the effect on the efficiency. The result is plotted in Fig. 9 for a typical turbocharger compressor and identifies that the actual polytropic efficiency in turbocharger compressor stages at low-speeds is substantially higher than the apparent value due to the effect of the heat transfer, by more than 10% at a tip-speed Mach number of 0.6. Corrections of the adiabatic efficiency for this effect show that the drop in efficiency with the rotational speed at low-speeds is overestimated by conventional measurements, and that, in fact, the actual polytropic efficiency stays nearly constant when the speed of the turbocharger decreases.

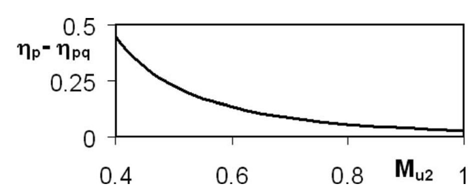

Fig. 9 Correction for the heat transfer effect on the apparent efficiency for a typical turbocharger compressor

5 Discussion and Outlook

A similar type of polytropic analysis can probably be formulated for the analysis of condensation and evaporation processes, where the latent heat causes a change in entropy; see, for example, Ref. [18]. The equations derived above are derived for a compression process, and for completeness the equivalent form of Eq. (29) for the turbine case becomes

$$v = \frac{(n-1)/n}{(\gamma-1)/\gamma} = \zeta_q + \eta_p = \eta_{pq} \tag{47}$$

6 Conclusions

Efficiency definitions for a compression process with work input and heat transfer have been analyzed. The isentropic analysis is shown to be flawed for a diabatic flow due to the fact that the hypothetical perfect process is reversible and adiabatic. A polytropic analysis is more useful for a diabatic process as it considers the changes between the actual end states of the real compression path. The polytropic analysis provides a simple way of taking heat transfer during the compression process into account by means of an additional heat transfer coefficient ζ_q in addition to the usual polytropic efficiency.

These issues are important both from a purely academic view, not only as an exercise in concepts in thermodynamics, but also as an important practical guideline to correctly defining the efficiencies. The practical utility of the approach is demonstrated by the analysis of heat transfer in a turbocharger compressor at low-speed. The analysis of measured performance suggests that the approximation of a constant heat transfer rate at low-speed is reasonable. This allows the apparent loss in efficiency due to the heat transfer to be estimated. Performance maps for typical radial compressors in turbochargers can then be recalculated with the corrected efficiencies that account for the heat transfer, and these are found to be much larger than previously reported.

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Nomenclature

a = speed of sound (m/s)
c = absolute flow velocity (m/s)
c_p = specific heat at constant pressure (J/kg K)
c_s = slip velocity (m/s)
D_2 = impeller tip diameter (m)
h = specific enthalpy (J/kg)
j = specific dissipation work (J/kg)
$\dot{m}$ = mass flow rate (kg/s)
M_{u2} = tip-speed Mach number
n = polytropic exponent
p = static pressure $(\mathrm{N/m^2})$
q = specific heat transfer (J/kg)
$\dot{Q}$ = heat transfer rate (J/s)
R = gas constant (J/kg K)
s = specific entropy (J/kg K)
T = temperature (K)
u_2 = impeller blade tip speed (m/s)
v = specific volume $(\mathrm{m^3/kg})$
w_t = specific shaft work (technical work) (J/kg)
y = specific flow work (or polytropic head rise) (J/kg)

Greek Symbols

β = relative flow angle (deg)
γ = isentropic exponent
η = efficiency
λ = work input coefficient
ψ = pressure rise coefficient
ϕ = flow coefficient
ν = polytropic ratio
ζ = heat transfer coefficient $(\mathrm{J/kg\ m^3\ K})$

Subscripts

1 = inlet conditions
2 = outlet conditions
Euler = from Euler equation
i = isothermal
irrev = irreversible
p = polytropic
q = diabatic/with heat transfer
s = isentropic
t = total

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